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A BRIEF COMPARISON BETWEEN MECHANICAL ASPECTS AND CONSTRUCTION OF ARCH BRIDGES DURING THE XVIIIth AND XIXth CENTURIES

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Abstract. *The aim of this paper is to compare the development of theoretical research on the collapse analysis of arches and vaults, with some significant constructions of arch bridges, in French and Italy during the XVIIIth and XIXth centuries. On this subject, the authors would develope a brief outline of most important researches about mechanical aspects of the arch bridge theory in the same centuries. Then it will be developed some considerations on the construction, behaviour and assessment of a little number of significant arch bridges, to verify the corresponding between construction, theoretical and mechanical approach, collapse mode and conservation approach of these architectures.*

1 INTRODUCTION

The aim of this paper regards a brief comparison between mechanical aspects and construction of arch bridges during the XVIIIth and XIXth centuries, particularly in France area. The literature on construction bridges in France is great and we can have a complete panoramic of most important bridges built in these centuries. From the *Traité des ponts* by E.-M. Gauthey[i] - edit by his nephew Louis Navier - to the *summa* of Sejournè on *Grandes Voutes*[ii] - published in 1913-16 - all the authors who studied bridges' construction have been offered an exhaustive *compendium* of mechanical theories, construction aspects, history, catalogue of this important branch of construction history.

2 EQUILIBRIUM AND LIMIT ANALYSIS OF ARCHES

The mechanical aspects of collapse bridges comportment were studied in these years by two point of view: whether theoretical (Sinopoli *et alii*, 1997) [iii] or technical and experimental aspects (Melbourne, 1995; 1998) [iv]. The most important evidence of this statement are the arch bridges proceedings from the first edition in Bolton (1995) to the present Symposium.

Starting from the Heyman's studies on *Stone Skeleton*[v], some researchers have been analysed collapse methods of arches and vaults from these hypotheses: 1) the masonry arch is assumed as a mechanical rigid system, subject to friction and unilateral constraints; 2) masonry has no-tensile strength; 3) the compressive strength is effectively infinite. Then, starting from these hypotheses according to Heyman's approach limit analysis, it has been demonstrated (Sinopoli *et alii*, 1998)[vi] that failure may be occur only when a sufficient number of hinges is formed to transform the arch into a collapse mechanism. An important consequence of this result is that safety depends by the existence of some thrust line completely lying within the ring of the arch. So, within the rigid-plastic theory, the "kinematic" theorem is generally used to evaluate "from above" the collapse load. In this theory approach a particular importance is assigned to the friction problem. In fact, according to the plastic theory, collapse mechanism with sliding in presence of Coulomb friction, cannot be treated for non-standard results. Then, friction results are not associated flow rule that invalidates the general bounding theorems.

The methodological approach considers that complete and correct results can be directly obtained investigating the equilibrium of a rigid arch, through the principle of virtual work by a "static" formulation. This "static" approach is based on determination of limit values of the admissible internal forces, and it is opposed to the "kinematic" approach, which determines the collapse mechanism without considering internal reactions.

As Coulomb stated, the static approach, re-discovered by Heyman in his book of Coulomb's *Essai* (…)[vii], concerning the equilibrium of the whole arch, defines a Coulomb's criterion, where safety corresponds to the existence of a common range of solutions of two inequalities for rotational equilibrium and for sliding equilibrium as following:

$$\text{Maximum } (H^{r,e}) \leq H^r \leq \text{Minimum } (H^{r,i}) \qquad \text{rotational equilibrium}$$

$$\text{Maximum } (H^{s}{,}_{min}) \leq H^r \leq \text{Minimum } (H^{s}{,}_{max}) \qquad \text{sliding equilibrium}$$

where quantities are the next significance. It is necessary to quote that with reference to rotational equilibrium, Coulomb's criterion is not correct, because it gives the necessary but generally not sufficient condition.

The Coulomb's solution was re-examined by Persy's static formulation[viii], where the equilibrium of the whole arch, comparing the extreme values of thrusts preventing rotation and sliding, is summarised by these inequalities as $L \leq H \leq l$ for thrust H applied at the extrados, and $L' \leq H \leq l'$ for thrust H applied at the intrados. Each value for L, l, L', l' refers to a different rotational collapse mechanism. Persy's criterion on safety corresponds to the simultaneous fulfilment of the two independent inequalities. It is important to note that this criterion gives the sufficient but generally not necessary condition for equilibrium.

The solution developed by Sinopoli *et alii*[ix] on the equilibrium of the arch considered as a voussoir of increasing width, using the principle of virtual works, defines the equilibrium condition of the voussoir for some virtual displacement. The necessary and sufficient condition for equilibrium of whole arch is the existence of a range common of sliding value of the thrust and the rotational value of the thrust. Then it is possible to observe that equilibrium and stability depend only by arch geometry.

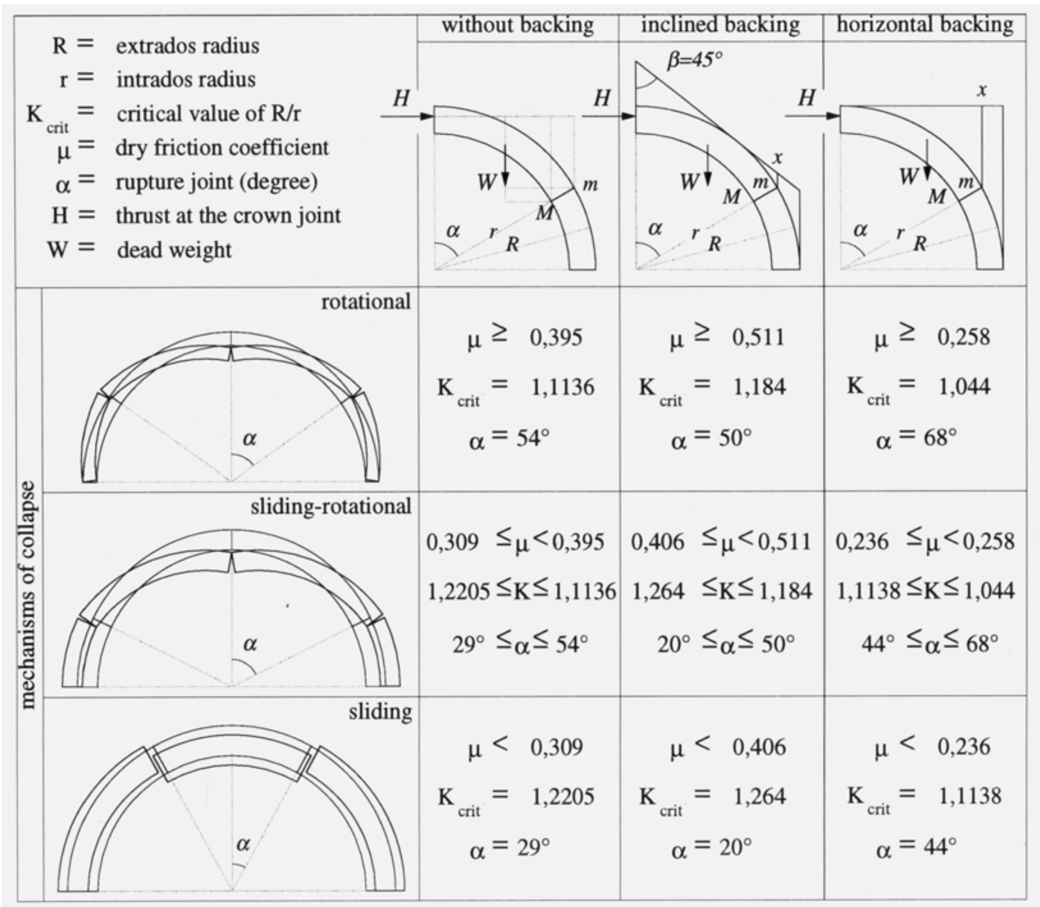

Figure 1: Mechanics of collapse of semicircular arches with constant thickness.
(From A. Sinopoli, M. Corradi, F. Foce: 1999).

A synthesis of these results is presented in the figure 1, where are represented some collapse mechanism of semicircular arches with constant thickness.

3 STRENGTH OF MATERIALS FOR MASONRY ARCH

Strength of materials related to the equilibrium of arch is quoted in a complete analysis in Navier' notes to the Gauthey's *Traité des ponts*. The most important statement quoted by Navier is that the thrust must be applied on the superior third medium of transversal section of the arch at the crown, and on the inferior third medium of transversal section of the arch at the joint of rupture. This condition establishes that the neutral axis takes one's places tangential to the section and so in masonry it is not possible to verify tensile strength. This restrictive condition is necessary if materials are not resistant to traction (N.T.R. materials), but this assumption is not generally verified, when materials have a resistance of tensile stress equal to 1/10 of their compression resistance. So, it is possible to admit that neutral axis is internal to transversal section, so that mortars are solicited to the limit of their tensile resistance, then before the crack.

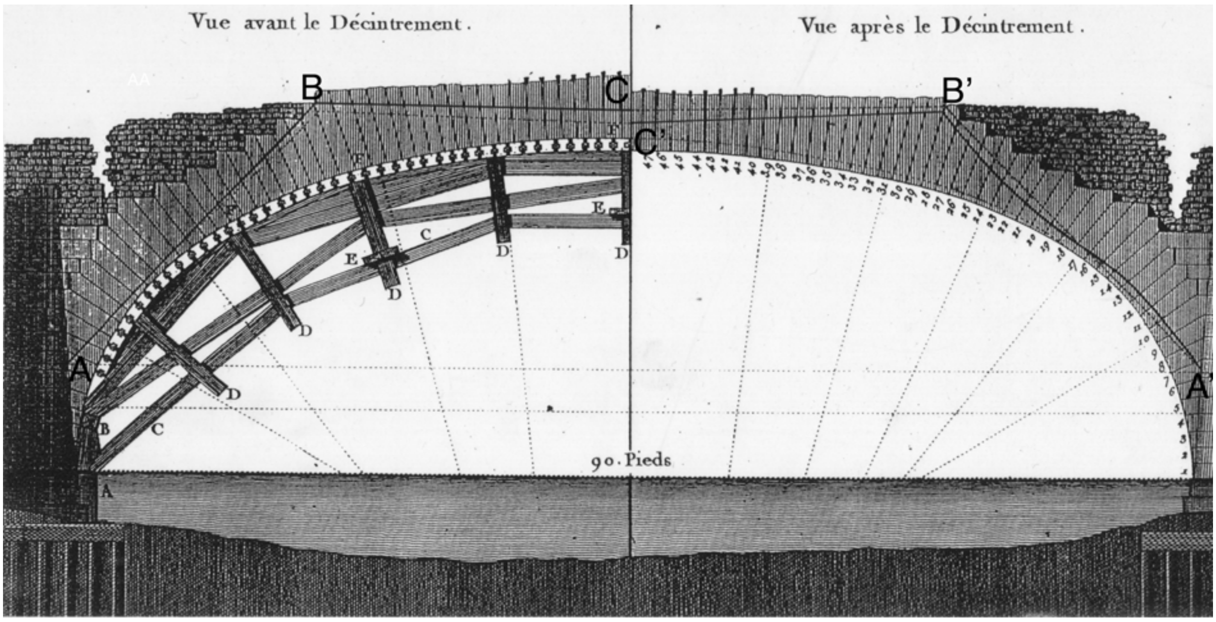

Figure 2: From Perronet's treatise. *Arche St. Edme de Nogent sur Seine construit en 1768.*

This condition is most important to evaluate the comportment of arches before and after the scaffolding take down, as cited by Perronet in his *Description des projets et de la construction des Ponts de Neuilly, de Mantes, d'Orléans et autres, du projet du canal de Bourgogne pour la communication de deux mers par Dijon et de celui de la conduite des eaux de l'Yvette et de Bièvre à Paris* [x].

In the Figure 2 is showed the arch of St. Edme de Nogent sur Seine, built in 1768. In this figure is clear the movement of the bridge "avant le Décintremet" and "après le

Décintrement". The line ABC on the left of picture, and the same line A'B'C' on the right, show arch deformation (see line B'C' with two different bending and variable bend radius). In the same figure, on the left and right, in correspondence of spandrels or backings, is the most evidence example of strength of materials - stones and mortars - subjected to tensile action, and of the consequent masonry crack. Perronet shows also that the fracture appeared in the backing confirms the collapse mechanism represented in Figure 1.

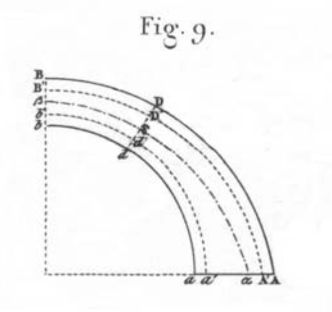

Figure 3: From Méry's paper. Range of admissible line thrust.

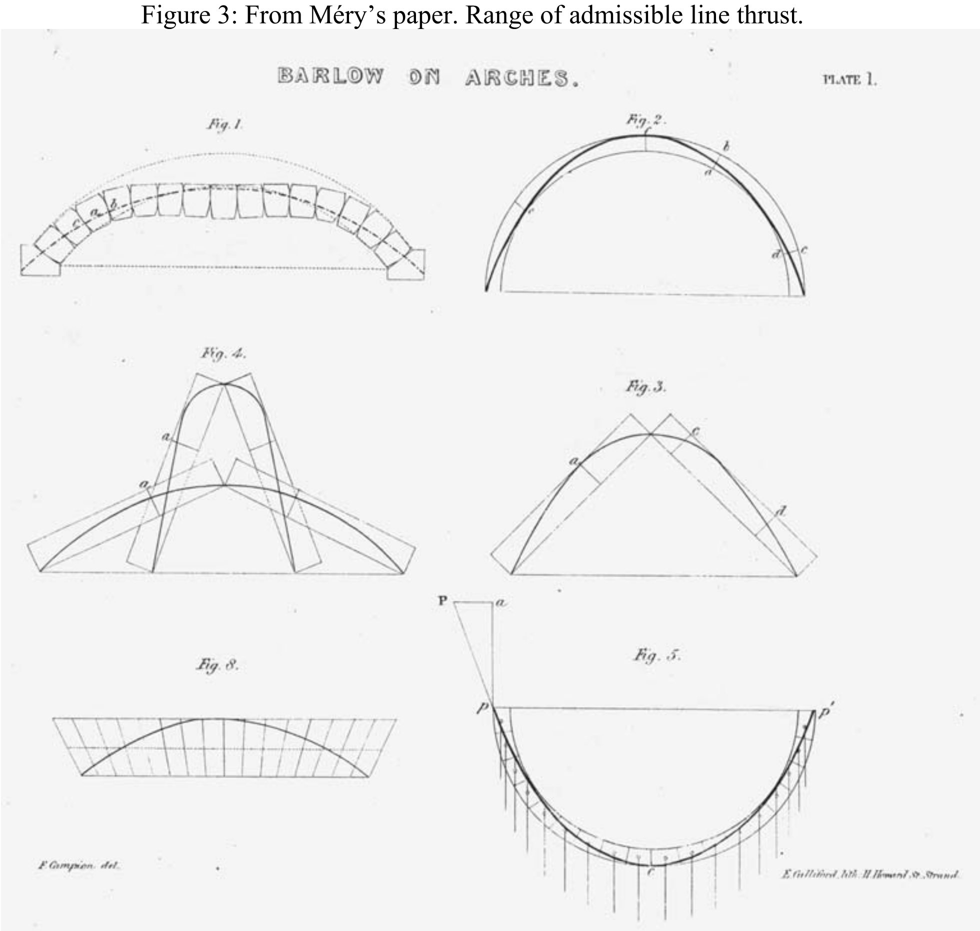

Figure 4: From Barlow's paper. Line thrust.

Perronet, and after him many arch bridges builders, evaluated this problem with more attention, to estimate the comportment of strength materials. Méry, in his note published in 1840, quoted that the neutral axis must be comprised in a range of values which establish a safety area, capable to guarantee the arch resistance. In this assumption "*la question de la solidité des voûtes est ramenée à celle de leur équilibre mathèmatique*" [xi], then mathematic and static solution must be converging to the same result.

In this case, the solution proposed by Navier appears as a possible equilibrium configuration before the formation of collapse mechanism for no-tension resistance materials. This solution, on the contrary, is a great approximation for materials capable to bear traction stresses, because after first cracks in mortar or masonry - if collapse mechanism as not still started - transversal joints cracked reduce the resistance area, increasing stresses and strains.

The question if joints are capable to bear increasing load, just to reach rupture tensile stress, defines the range of admissible values of loads before the formation of collapse mechanism. It is a question of great importance, because in this case it is possible to define criteria of safety for masonry arches. In 1846 William Henry Barlow published a paper "On the existence (practically) of the line of equal Horizontal Thrust in Arches, and the mode of determining by Geometrical Construction".

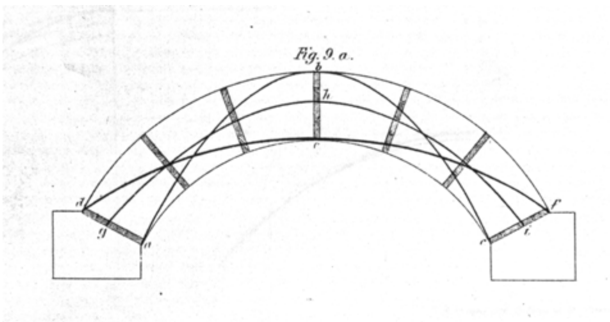

Figure 4: From Barlow's paper. Range of line thrust.

Figure 5: Line thrust of equilibrium (model).

The Barlow's supposition on the existence of a certain curve or line throughout the voussoirs of an arch – derived by theory of catenaria equilibrium, showed by David Gregory in 1697 – defines a set of possible lines thrust, according to the comportment of arches under loads. As it is showed in Barlow's plate number 1 and number 2, it is possible to evaluate a line thrust which is into arch thickness. Barlow's results correspond to a set of possibilities of arch equilibrium, before the formation of collapse mechanisms; they are compatible with strength of materials. The subject is one of great difficulty before it embraces many considerations. The arch is not homogeneous, but it is composed of separated voussoirs, bound together by cement; it involvs other principles as elastic mass of mortars. The pressure in keystone may be found to the wall surface, from the extrados to the intrados of key. Then the "line of pressure" describes the condition of equilibrium of arch, in the range of admissible values lies to strength of materials (Navier and to the extreme Barlow), or kinematic collapse mechanism (Heyman). The question refers whether to strength materials or kinematic collapse mechanism, and it is still open, while the history of structural applied mechanics to the arches shows different points of views.

12 CONCLUSIONS

The two "souls" of this important question: a) the analysis of masonry arches by collapse methods; b) the definition of line thrust (*Das Rätsel der Architektur*[xii]) and then the strength of materials, are complementary for the comprehension of vaults comportment under static and dynamic loads. The question is still open, and many studies are devoted to define some possible lines of research, to limit the domain of strength or that of collapse. Certainly, the collapse mechanism method appears the nearest to the reality, as showed in Figure 4, where we can observe Vignole's bridge under collapse, successively to the rotational action of one pillar (ground subsiding under the pillar). The historical analyses have been showed that it is possible to revisit the arch and vaults theory developed in XVIIth and XIXth centuries, to increase the knowledge in this important topic of structural applied mechanics[xiii].

Figure 4: Vignole's Bridge under collapse.

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