RELIABILITY-BASED ASSESSMENT OF A CONCRETE ARCH BRIDGE
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Key words: Reliability, Pushover, Redundancy, Model updating, Arch Bridges

Abstract. The paper deals with the safety evaluation of a relatively old concrete arch bridge. On this respect, first, a finite element model of the structure was opportune calibrated through modal information desumed by dynamic tests performed at the site. Then, two different non linear limit analyses were performed on the calibrated model. The first analysis tends to evaluate structural redundancy and resistance to vertical load uniformly distributed on the superstructure. The second analysis is devoted to evaluation of structural performances with respect to lateral seismic-type loads. The evaluated structural resistances were then compared with the internal stresses induced by actual traffic loads and maximum expected earthquake, obtaining a global evaluation of the bridge structural safety.

1 INTRODUCTION
Maintaining a bridge network requires clearly defined performance criteria, a set of corresponding condition indicators or ratings, defined rules for structural assessment, global and local damage descriptions with rates of deterioration, maintenance-work prioritization, and predictions of future deterioration. Various approaches have been developed to satisfy each of these requirements for realization of Bridge Management Systems (BMSs). Currently available BMS’s, including the earliest Pontis\(^1\), BRIDGIT\(^2\), the Finnish\(^3\) and Danish\(^4\) BMS’s, rely primarily on information obtained through visual inspections. In these BMS’s, dedicated functions help visual inspectors to describe the bridge condition at the system- or component-level and to record condition ratings, which are quantified and standardized through a priority ranking procedure\(^5\). These methods have the advantage of relying on traditional inspection methods in which bridge inspectors have already been trained and with which they are comfortable and familiar. Additionally, the ranking procedures are suitable for implementation in more general BMS frameworks and they are easily linked with maintenance and rehabilitation actions and costs\(^6\). The methods have been considered for
many years the most natural way to collect and organize data based on visual inspections, the
most cost-effective means for assessing bridge safety.

However, developers of the next generation of BMS’s are looking to base designs on lifetime reliability and whole-life costing. A reliability-based system permits the use of reliability index that is theoretically well-founded indicator of safety and serviceability. Combining reliability of different failure modes with deterioration model, a reliability profile for the observed bridge can be obtained. Based on a prescribed target reliability level, the probability distribution of the expected time of intervention (i.e. maintenance, repair) can be evaluated\textsuperscript{7,9}.

The present paper presents the results of preliminary efforts in evaluating structural safety and reliability of an existing arch bridges belonging to the road network of Province of Teramo, Italy, currently supervised by a simple BMS developed by DISAT, University of L’Aquila. On this respect, recent research studies have followed the idea to define a complete framework to asses infrastructure condition through structural identification methodology that contains: experimental and analytical arts, information technology, decision making arts and non-technical issues\textsuperscript{10,11}. It is evident the needs to use all the information available on the infrastructure (visual inspections, measurements, modal analysis, instrumented monitoring, bridge database, etc…) to making the correct decision with respect the reliability evaluation for service and safety and the overall bridge management.

On this scenario, a possible methodology is proposed which it permits to verify if the structure is able to absolve its principal functions towards actual loads, and to estimate its last reserves of resistance. The used methodology is illustrated through a case study (Villa Passo Bridge) and it constitutes a preliminary approach to build up a more general framework to assess bridge condition through the fusion of different mechanism of evaluation as visual inspection procedures\textsuperscript{5} and reliability evaluations\textsuperscript{7,9}.

The Villa Passo bridge is a concrete arch bridge constructed on 1917 to overpass the deep valley of Salinello river. On 1999 have been fully retrofitted with some main modification in the structural system. Based on the results of visual inspections and recursive dynamic testing\textsuperscript{12} a reliability based assesment of the bridge has been performed. Two different non-linear limit analyses have been performed. The first one consists in performing a vertical pushover analysis, through a step by step procedure, up to the collapse of the structure, recognizable from the formation of a sufficiently high number of plastic hinges. The second analysis consists instead in a lateral pushover analysis. The values determined through the two analyses permits to numerically evaluate the last resistance of the structure in vertical and lateral direction. These values can be, then, compared with the internal stresses (or related displacements) induced by the actual traffic overloads and caused by the maximum earthquake attended in site, obtaining so a global evaluation of the safety of the structure.

2 RELIABILITY EVALUATION OF A BRIDGE STRUCTURE

The reliability of a single bridge component, can be evaluated through its probability of failure $P_f$, as defined:

$$P_f = P[R - S < 0]$$  (1)
where $R$, $S$ are respectively the last resistance of the element and the internal stresses due to loads. $R$ and $S$ in the eq.(1) are independent random variables, having a certain level of uncertainties that can be reduced, but not eliminated, with some investigations in site. The safety of a structure is generally expressed through the index of reliability $\beta$. If $R$ and $S$ are standard normally distributed (zero mean and unit standard deviations), $\beta$ can be expressed as

$$ \beta = \Phi^{-1}(1 - P_f) $$

where $\Phi^{-1}$ is the inverse function of the normal standard distribution function.

### 2.1 Reliability of a single component of the structure

It is assumed that the resistance $R$ and the stress $S$ to which the element is submitted, are random variable with lognormal distribution. Therefore, the reliability index can be determined through

$$ \beta_{member} = \frac{\ln(R/S)}{\sqrt{\sigma_R^2 + \sigma_S^2}} $$

where $\overline{R}$ and $\overline{S}$ are the mean of the resistance and the stress while $\sigma_R$ and $\sigma_S$ are respectively the standard deviations of $R$ and $S$.

### 2.2 Reliability of the substructure (superstructure)

The reliability of a bridge substructure is evaluated in a conceptual equivalent way to the case of a single element of it. Assuming again that the loads $F_u$ and $F_w$ are random variables following lognormal distributions, the reliability index of the substructure against the limit state can be defined as

$$ \beta = \ln\left(\frac{F_u}{F_w}\right)/\sqrt{\sigma_{F_u}^2 + \sigma_{F_w}^2} $$

where $\overline{F_u}$ represent the mean of the load causing the failure of the substructure (superstructure) while $\overline{F_w}$ is the mean of the maximum expected external load occurring during the life of the structure and $\sigma_{F_u}$ and $\sigma_{F_w}$ are respectively the standard deviations of $F_u$ and $F_w$. In redundant structures $\beta_{ult}$ is always greater than $\beta_{member}$; the difference is due to the structural reserve of carrying loads after the failure of the first element. In other words the difference $\Delta\beta$ constitutes a direct measure of the structural redundancy and it can be calculated by

$$ \Delta\beta = \ln\left(\frac{F_{ult}}{F_{member}}\right)/\sqrt{\sigma_{Fult}^2 + \sigma_{Fmember}^2} $$

### 2.3 Redundancy of bridge structures

Redundancy is the capability of a bridge structural system to carry loads after damage or failure of one or more of its members. It can be evaluated through direct redundancy analysis executed by a finite element model of the structure. Two different analysis can be conducted depending on the directions of the prevailing loads.
Under vertical overloads, since they represent the most frequent cause of collapses of the bridge superstructures, the evaluation of the redundancy can be achieved through a pushover analysis. Indeed, the failure of a member of the structure will be recognizable by the formation of a series of plastic hinges, while the failure of the whole superstructure will be identified from the formation of the number of plastic hinges that determines a collapse mechanism. The ratio between the vertical force causing the failure of the entire system and the force causing the failure of one structural member is defined as the system reserve ratio for the ultimate capacity. It constitutes a direct measure of the structural redundancy.

For lateral overloads such as earthquakes, wind, stream flow or accidental collisions of vehicles or vessels the pushover analysis is similar to the previous one, the main differences are determined by the positioning of possible plastic hinges, that should be provided by the analyst in some codes. A ratio between the lateral force causing a global bent failure and the force causing the failure of one structural member constitutes the redundancy measure.

3 RELIABILITY EVALUATION OF THE VILLA PASSO BRIDGE

3.1 Dynamic spectral properties identified through EFDD

A modal identification procedure based on an ambient excitation has been applied to the bridge structure13 (length spans in meter: 21.6 - 22.0 - 60.4 - 20.0). The ambient excitation was given by the regular traffic on the bridge. The identification of the modal parameters was carried out using 10 accelerometers: 14 points of the bridge deck were monitored on 2 different setup to identify the vertical components (see fig.1a and 1b) while 5 points were used to identify the horizontal ones in the 3rd setup (see fig.1c). Time series corresponding to about 3000 first natural period of the structure were acquired at a sampling rate of 400 HZ.

The used identification procedure is based on the Enhanced Frequency Domain Decomposition and was conducted using the software ARTeMIS14 developed at the Aalborg University. Even if the procedure is able to identify, in principle, all the modes of the continuous structure, because of the usually not-large number of available accelerometers, the attention was focused on the first five structural modes (3 vertical and 2 horizontal modes) clearly identifiable with the used instrumentation setup.

![Fig. 1. The accelerometers lay-outs: setup 1 (vertical), 2 (vertical) and 3 (horizontal).](image)
In Fig.2 the spectral density functions for the two considered setups are reported in the meaningful frequency ranges. Three and two spectral lines are easily recognizable respectively and correspond to the first two vertical modes (V1 and V2 in Fig.2a), first torsional (T1 in Fig.2a) and to the first two horizontal modes (H1 and H2 in Fig.2b). For each identified frequency the corresponding modal shape has been evaluated and reported in Fig.3. After identifying each mode in the frequency domain, going back in the time domain it is possible to estimate the relevant modal damping by means of the logarithmic decrement method. The identified natural frequencies and damping ratios are reported in Table 1.

**Fig. 2.** The peak peaking on the spectral density functions: a) vertical modes, b) horizontal.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st vertical</th>
<th>2nd vertical</th>
<th>1st torsional</th>
<th>1st horizontal</th>
<th>2nd horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ (Hz) (case A)</td>
<td>3.67</td>
<td>5.64</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\xi$ (%) (case A)</td>
<td>1.5</td>
<td>1.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f$ (Hz) (case B)</td>
<td>3.66</td>
<td>5.63</td>
<td>9.43</td>
<td>1.58</td>
<td>2.65</td>
</tr>
<tr>
<td>$\xi$ (%) (case B)</td>
<td>1.6</td>
<td>1.1</td>
<td>0.8</td>
<td>1.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>
3.2 Villa Passo Bridge FE model updated by modal testing

A large finite element model has been built up (Fig.4) and used to enhance the knowledge of the dynamic behaviour of the arch bridge. A convergence analysis was conducted starting with a selected FE model with initial frequencies ($f_{0,1v} = 3.70158$ Hz, $f_{0,2v} = 5.57684$ Hz).

The analysis has been performed with a recursive increment of the number of finite elements, preserving the initial mesh and subdividing both the shell and the beam element in equal numbers. The results of the analyses are reported in Fig.5a, where the convergence error $\varepsilon = (f_{oi} - f_{ci})/f_{oi}$ is reported for the two first vertical frequency as function of the number (N) of dofs involved in the analysis. The analysis permits a prediction of the exact solution at higher frequencies that the ones obtained with low order model; this is due mainly to the lumped mass assumption in the FE model construction. The expected relative error is of the same order of the error evaluated by updating the material properties. (0.4% for $f_{1v}$, and 0.7% for $f_{2v}$ are the error obtained by an augment of one order (10×) in the model dofs). A preliminary study has been conducted on the sensitivity of the first two vertical natural frequencies ($f_{1v}$, $f_{2v}$) to the modification of two selected material parameters (Young modulus $E$, material mass density $m$). The results, summarized in Figs.5b and 5c, have been used for a manual calibration of the FE model.

![Fig. 4. Villa Passo Bridge: aspects involved in the analytical art: a) visualizing, b) conceptualizing and geometric modelling, c) mechanical modelling (finite element mesh)](image)

![Fig. 5. Frequency sensitivities of 1st and 2nd vertical mode: a) fem convergence analysis; b) c) effects of concrete Young modulus and mass density on the two eigenfrequencies)](image)
The comparison between the selected FE model candidate and the modal testing data have been conducted evaluating the estimate relative error $\varepsilon_{ei} =$ $(f_i - f_{ei})/f_i$ between the frequencies $f_i$ evaluated by FEM and the frequencies $f_{ei}$ estimated by EFDD. A direct measure of the error in the modal shape was pursued using the conventional MAC defined as

$$MAC(\phi_i, \phi_f) = \frac{\sum_{i=1}^{n} \phi_i^H \phi_f}{\left(\sum_{i=1}^{n} \phi_i^H \phi_i\right)^{1/2} \left(\sum_{i=1}^{n} \phi_f^H \phi_f\right)^{1/2}}$$

where $\phi_i$ are the FEM modal shapes and $\phi_{ei}$ are the estimated ones. A summary of the achieved precision is presented in Table 2 for the two in-situ testings conducted on the bridge.

<table>
<thead>
<tr>
<th>Mode</th>
<th>EFDD $f$ (Hz)</th>
<th>FEM $f$ (Hz)</th>
<th>$\varepsilon_e$ (%)</th>
<th>$\varepsilon_c$ (%)</th>
<th>MAC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st vertical (case A)</td>
<td>3.67</td>
<td>3.69083</td>
<td>0.56</td>
<td>0.44</td>
<td>0.994</td>
</tr>
<tr>
<td>2nd vertical (case A)</td>
<td>5.64</td>
<td>5.54924</td>
<td>-1.63</td>
<td>0.69</td>
<td>0.981</td>
</tr>
<tr>
<td>1st vertical (case B)</td>
<td>3.66</td>
<td>3.69083</td>
<td>0.83</td>
<td>0.44</td>
<td>0.991</td>
</tr>
<tr>
<td>2nd vertical (case B)</td>
<td>5.63</td>
<td>5.54924</td>
<td>-1.45</td>
<td>0.69</td>
<td>0.973</td>
</tr>
</tbody>
</table>

### 3.4 Nonlinear static limit analysis to vertical loads

A preliminary linear analysis has been performed on the structure to determine the internal stresses due to common vehicular loading. Applying on the structure opportune vehicular loads, determined with a traffic analysis, the stresses in critical sections of the structural elements has been determined. The results are summarized in Fig.6.

Successively, positions of plastic hinges are assigned to the critical sections of the beams belonging to the superstructure and a vertical load uniformly distributed is increased at each step up to reach the failure condition for the superstructure.

<table>
<thead>
<tr>
<th>SECT. 1</th>
<th>$M_1 = -81,67$ kNm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_1 = -97,5$ kN</td>
</tr>
<tr>
<td>SECT. 2</td>
<td>$M_2 = 65,52$ kNm</td>
</tr>
<tr>
<td></td>
<td>$V_2 = 103,35$ kN</td>
</tr>
<tr>
<td>SECT. 3</td>
<td>$M_3 = -80,32$ kNm</td>
</tr>
</tbody>
</table>

Fig. 6. Stresses induced by the traffic loading: a) position of the critical section, b) values of internal stresses
Figure 7 shows two steps of the sequence of plastic hinges formation associated with the amplitude of the vertical load. Fig. 7a represents the 1st plastic hinge formation while Fig. 7b represents an intermediate scenario before the global collapse of the superstructure. According with the method presented in the previous sections the structural reliability was evaluated by Eq. (4) where the standard deviations $\sigma_{\text{member}}$, $\sigma_{\text{ult}}$ were assumed as suggested by analyses conducted by governative institutions\(^1\) (\(\sigma_{\text{member}} = 0.13\), $\sigma_{\text{ult}} = 0.30$).

Table 3 shows the reliability indices of the superstructure, for bending or shear, at the beam supports (sects. 1 and 3) and at the middle span (sect. 2) (Fig 6). These indices are related to the first plastic hinge formation in the bridge deck belonging to the superstructure.

<table>
<thead>
<tr>
<th>Sect.</th>
<th>Member</th>
<th>Bending</th>
<th>Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sect. 1</td>
<td>$\beta_{\text{member}}$</td>
<td>2.93</td>
<td>2.35</td>
</tr>
<tr>
<td>Sect. 2</td>
<td>$\beta_{\text{member}}$</td>
<td>4.13</td>
<td></td>
</tr>
<tr>
<td>Sect. 3</td>
<td>$\beta_{\text{member}}$</td>
<td>3.02</td>
<td>2.35</td>
</tr>
</tbody>
</table>

Fig. 8. Reliability indices at the critical sections; effects of structural redundancy on reliability to vertical load.

Increasing the loading in the pushover analysis, several hinges reaches the yielding limits in both the superstructure and in the arch beams. Assuming as incipient collapse, the new scenarios at each step the increment of reliability showed in Fig. 8 can be reached that permits to evaluate the structural redundancy with respect to vertical loads. The analysis are considered finished when the structure is not able to carry any increment of push loading and the increment $\Delta \beta$ in the reliability index reaches a stationary value.
3.5 Nonlinear static limit analysis to horizontal loads

In order to perform a non-linear lateral limit analysis of the whole structure, a few modifications of the finite element model was necessary. Indeed, in order to assign discrete plastic hinges, the concrete walls forming the piers, previously modelled with shell elements, were modelled in these analyses with frame elements connected by rigid links with the secondary beams of the bridge (Fig. 9a). Soil-structure interaction was also modelled through horizontal and rotational springs. A Modal Pushover Analysis (MPA) was performed, in which the structure is controlled by a mode collapse shape unchanged after yielding.

In the analysis, firstly, the structural weight were applied and a vertical nonlinear analysis was performed including both local (P-δ) and global (P-∆) nonlinear geometrical effects. Then, the horizontal load was increased through the MPA and the global capacity curve (base shear versus top displacement) was obtained.

To evaluate the structural reliability to seismic loads, bridge lateral resistance capacity was compared with the expected stresses induced by seismic force. In particular, the latter evaluation was performed according to linear response spectrum presented in the recent Italian code. Consequently, the amplitude of the first horizontal modal load was easily evaluated as 

\[ F_w = M S_d(T_i) \]

where \( M \), \( T_i \) and \( S_d(T_i) \) are the modal mass, the natural period and the response acceleration according to the design response spectrum, respectively.

Assuming a design ground acceleration \( a_g = 0.25g \) and B-type soil, the elastic spectrum was defined and through the structural factor \( q = 3.5 \), the design response spectrum permitted the evaluation of the acceleration response (\( S_d(T_i) = S_d(0.56) = 1.3 \)). Then, the seismic load \( F_w \) was easily evaluated (with \( U_y = 0.7, M = 0.7 M_i = 2227.2 kNm/s^2 \) \( F_w = 1.3 M = 2895 kN \)).

The reliability index referred to first member failure was evaluated according to Eq.(4) \( (\sigma_{F_w} = 0.13, \sigma_{U_y} = 0.33) \) where \( F_w \), determined through MPA, is the level of pushing force causing the first failure. The residual strength or redundancy \( \Delta \beta \) with respect to lateral loads was estimated increasing the lateral forces and evaluating \( \Delta \beta \) according to Eq.(5). The numerical results are depicted on Fig.9b where the augment of reliability due to structural redundancy \( \Delta \beta \) for each level of pushing force \( F_w \) (Nstep) is added to the reliability associated to the first member failure to obtain the global reliability index \( \beta_{ult} = \beta_{member} + \Delta \beta \).
4 CONCLUSIONS

A methodology for evaluation of structural safety of an existing concrete arch bridge has been pursued for the loading cases of vehicular traffic and expected earthquakes in site. The method utilizes as safety indicator the reliability factor $\beta$, the use of which permits to take into account the uncertainties related to the external loadings and the internal resistances with particular attention to the evaluation of the effects of the structural bridge redundancy. The study evidences that while the reliability related to the formation of the first plastic hinge is quite similar for vertical vehicular traffic and horizontal seismic loads, the vertical structural redundancy in sustaining traffic loading is significantly less than the one related to horizontal seismic loads. The presented reliability analyses constitute a preliminary investigation useful to conceive a reliability based bridge management system.

REFERENCES