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# STUDY ON STABILITY CRITERION OF SUPER-LONG CONCRETE ARCH

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**Abstract.** *The methods to verify the stability under static and seismic loads were proposed and its applications for a super-long span concrete arch bridge were discussed. The methods were essentially to account for the negative eigen values for the tangential stiffness matrices of a system and to get the seismic-buckling capacity indicators by means of the repeated eigen-analyses. Using these methods, the structural performances were investigated for a trial designed 600m center span concrete arch bridge. It was confirmed that the arch bridge has the sufficient seismic and stableness performance even in post-peak range without undergoing sudden failure according with the change of the deformation mode such as bifurcation.*

## 1 INTRODUCTION

There are trial designs of super-long span concrete arch bridges and they may be constructed in the near future. For example, Japan Society of Civil Engineers "subcommittee on design methods for long span concrete arch bridges" developed a trial design of a long span concrete arch bridges that have a center span length of 600m [i]. The concrete arch bridge construction in severe seismic zone is limited to the classes of over 200m center span due to the uncertainty for its aseismicity as well as its stability problem. Feasibility of arch bridges of the class of over 600m center spans, therefore, depends on its stability against local or global buckling of their systems. The stability should be checked not only in the static state but also during earthquake oscillation.

In this paper, theories developed to assure its stability are described and its applications for a 600m span concrete arch bridge are discussed. The theories are essentially to account for the negative eigen values for the tangential stiffness matrices of a system and to get the seismic-buckling capacity indicators by means of the repeated eigen-analyses. Using these theories, the seismic and the stableness performance are investigated for a 600m center span concrete arch bridge when the dynamic equations are solved in time stepwise considering both the material and the geometrical nonlinearlity

## 2 VERIFICATION METHOD OF STABILITY

To check the stability against local and global buckling of the structures, linear buckling eigen analysis by Eq. (1), which offers a buckling mode vector and the buckling load-factor (multiple factor for reference loads) with the smallest eigen value, is usually adopted.

$$\left([K]+\lambda\left[K_{g0}\right]\right)\{v\}=\{0\} \tag{1}$$

where $[K]$: elastic stiffness matrix, $[K_{go}]$: geometrical stiffness matrix for initial stress, $\{v\}$: buckling mode vector, $\lambda$: buckling-eigen value. Although the method is a simple one, it can be strictly used only for a buckling investigation under dead load (gravity) and it is available only in elastic and small displacement range. It's a technique with much restriction.

In this chapter, two kinds of method to verify the stability are described under static load and under seismic load considering the mixed (material and geometrical) non-linearlity.

### 2.1 Negative Eigen value for Tangential Effective Stiffness Matrix

An equilibrium state is called stable if the response on a vanishingly small disturbance also remains vanishingly small [ii]. A condition for a state of stable equilibrium under dead load is that

$$\{u\}^T\left([K_t]+\left[K_g\right]\right)\{u\}>0 \tag{2}$$

where $[K_t]$ and $[K_g]$ are tangential stiffness matrix and geometrical stiffness matrix at the arbitrary incremental steps, $\{u\}$ is nodal displacement vectors.

For a critical state of neutral equilibrium, it is defined that the tangential effective stiffness matrix($[K_t]$ +$[K_g]$) of a structure is singular or at least one eigen value( $\lambda_i$ ) is zero.

$$\det([K_t]+[K_g])=0$$
$$\prod_{i=1}^{n}\lambda_i = 0 \tag{3}$$

The structure lose the stability when the tangential effective stiffness matrix satisfy these conditions. The point which is defined by these equations is the critical points. The critical points are classified as limit point and bifurcation point. The limit point is maximum point or minimum point of the load-displacement relationship. Therefore, the structures is on the unstable equilibrium state in post-peak region after the maximum load point. On the other hand, the bifurcation point is defined as Eq.(4) with the normality condition of the eigen vector $\{v_1\}$ corresponding to the zero eigen value and the load vector$\{f\}$.

$$\{v_1\}^T\{f\}=0 \tag{4}$$

The bifurcation point is usually called as buckling point and new equilibrium paths with the different deformation modes from the one on fundamental path are emanated from a point. Therefore, the stability can be checked by the eigen values of the structure which are the positive and the negative in mixed non-linear analysis[iii]. This discussion is satisfied clearly in static analysis. In dynamic analysis, however, the tangential effective stiffness matrix is replaced by Eq.(5) and it does not show the negative eigen value for infinitesimal time since the term of mass and damping matrices become infinite.

$$[\overline{K}]=[K_t]+[K_g]+\frac{\gamma}{\beta\Delta t}[C]+\frac{1}{\beta\Delta t^2}[M] \tag{5}$$

where $[C]$: damping matrix, $[M]$: mass matrix, $\Delta t$ : incremental time, $\beta$ and $\gamma$ : parameters of Newmark's $\beta$ method. Therefore, the checking of the negative eigen values for $[K_t]$ + $[K_g]$ does not imply the necessary and sufficient condition in dynamic action, but it is sufficient condition for the stability. It is conservative evaluation for the stability, since the loss of the stability in dynamic action should occur after the loss of the stability conditon in the static loading. When a dynamic analysis is completed stably without any negative eigen values for all time steps, the structural system is regared to be stable against the supposed earthquake. It is noted that the absolute value of smallest eigen value do not indicate the degree of the dangerousness of buckling. The importance of these eigen values is in whether positive or negative. The method can provide limited information about buckling when buckling doesn't occur in an analysis.

### 2.2 Seismic-Buckling Capacity Indicators

We focus on a phenomenon just like buckling under the static loads, because it is the phenomenon of bringing about serious damage to a structure and it may be overlooked only by a mixed non-linear time history analysis. We try to apply the eigen-analysis by Eq.(6) in each time step of a dynamic analysis as shown in Fig.1, instead of Eq. (3).

$$\left([K_t]+\lfloor K_g \rfloor+\lambda\lfloor \Delta K_g \rfloor\right)\{v\}=\{0\} \tag{6}$$

where $[K_t]$: tangential stiffness matrix in the previous step, $[K_g]$: geometrical stiffness matrix in the previous step, $[\Delta K_g]$: incremental geometrical stiffness by the initial stress in the current step, $\{v\}$: buckling mode vector, $\lambda$ : buckling-eigen value. As a solving of the equation, eigen values and their paired buckling mode vectors can be obtained. The first (minimum) eigen value indicates the multiple factor for the stepwise incremental stresses of each member of the system to the nearest buckling point (static-unstable point in dynamic action)[iv] .

When the 'Inverse iteration method' is adopted as a buckling eigen analysis method, the first buckling eigen value $\lambda_1$ is minimum in absolute. So it can be negative. $0.0 \le \lambda_1 \le 1.0$ indicates that there is a buckling point within the time step interval. In the state of $\lambda_1$>1.0, it is getting near to the buckling point but the larger $\lambda_1$ is, the farther to the nearest buckling point. $\lambda_1$<0.0 means that it is getting far from the nearest buckling point.

In an incremental static analysis, Eq.(6) is applied only when the buckling point between continuous two steps is found out by checking the determinant value of the matrix. But in the dynamic analysis it should be carried out repeatedly at a fixed step interval because the significance to use this method is not only for the judgment of unstableness but also for the suggestion of the degree of the dangerousness for buckling with eigen values even while the system is stable. Time history of $\lambda_1$ indicates the dangerous time zone. And the minimum $\lambda_1$ in duration can be a performance indicator for buckling during supposed earthquake. It must be noted that when the minimum $\lambda_1$ of some cases are compared, integration-time-intervals ($\Delta t$) of dynamic analyses must be unified.

We knew the application of Eq.(6) is originally limited to the static analysis for conservative forces. But we consider it is available in each step of a mixed non-linear dynamic analysis approximately. Although dynamic oscillations such as earthquake motions are not totally conservative of course, we regard the incremental dynamic force of each time step as conservative because the stepwise incremental responses are accumulated (conserved) as well as in an incremental static analysis.

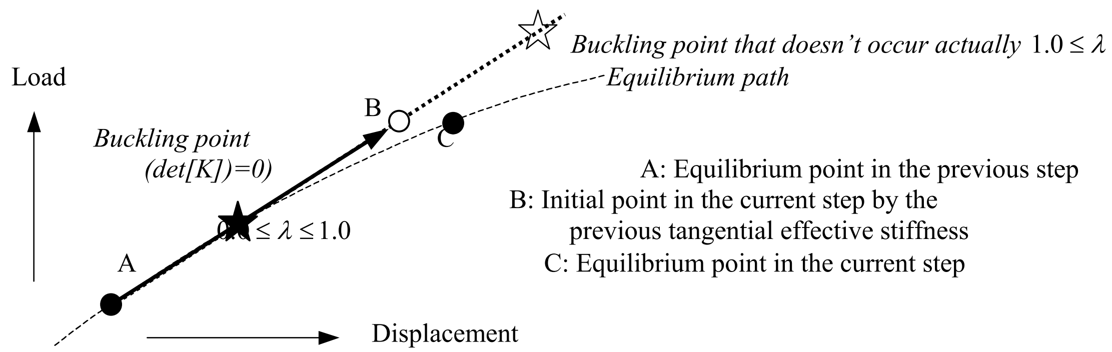


Figure 1: Buckling point in a step of incremental-static-analysis

## 3 A TRIAL DESIGNED 600M SPAN CONCRETE ARCH BRIDGE

Japan Society of Civil Engineers "subcommittee on design methods for long span concrete arch bridges" developed a trial design of a long span concrete arch bridge that have a center span length of 600m[1]. The model is an arch-under-deck Lohse fixed-arch bridge with a rise of 100m. Section forces under dead loads are calculated for all falsework construction methods. Under in-plane and out-of-plane seismic loads, the section forces are calculated by the three-dimensional elastic frame analysis in which the horizontal seismic coefficient ($k_h$) is assumed as 0.20. The section shapes are determined that the stresses of concrete and reinforcement caused by the section forces satisfy the allowable stress requirements. The material properties and the allowable stress are shown in Table 1.

The trial designed arch bridge with 600m center span which satisfy the design requirement is shown in Fig.2.

Table 1: Material Properties

| | Strength(N/mm$^2$) | Allowable stress(N/mm$^2$) | |
|---|---|---|---|
| | | dead loads | seismic loads |
| Concrete | 60 | 24 | 36 |
| Reinforcement | 685 | 180 | 685 |

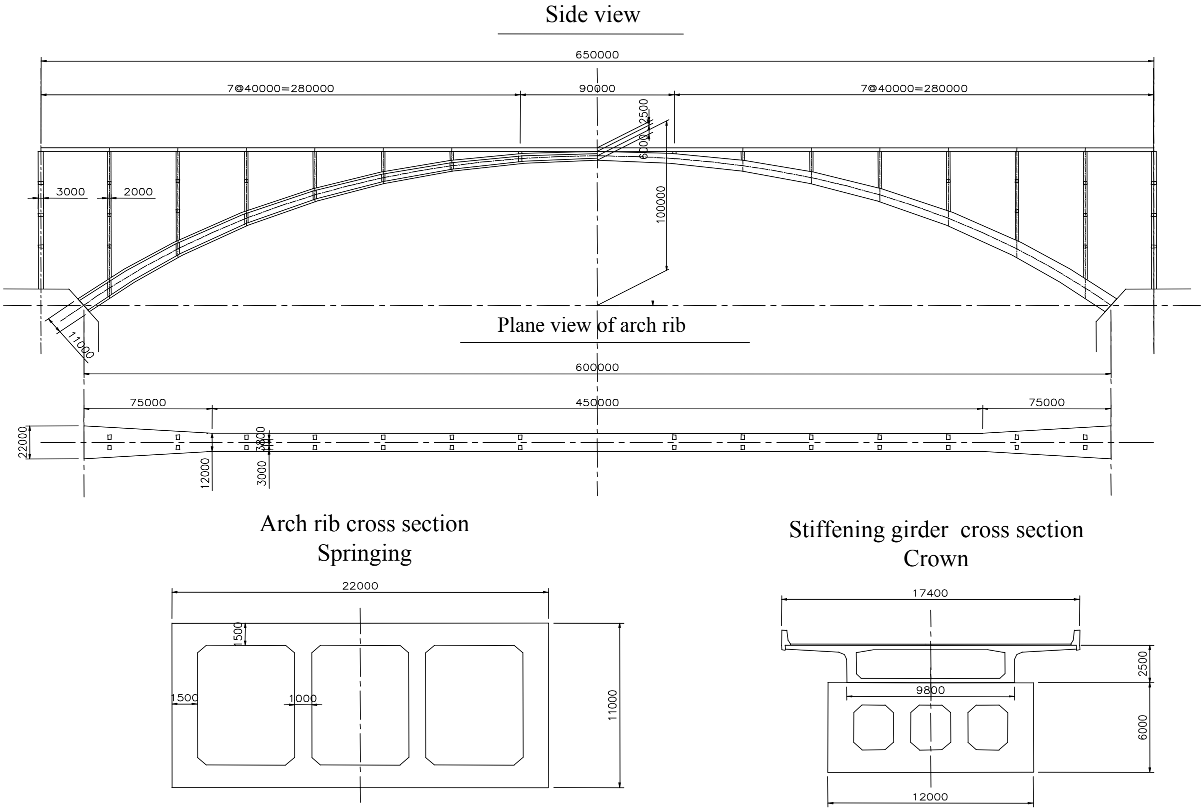


Figure 2: A trial designed 600m span concrete arch bridge

## 4 STABILITY VERIFICATION UNDER STATIC LOADS

In this chapter, verifications for static loads are carried out considering the evaluation of stability for a trial designed 600m span concrete arch bridge.

### 4.1 Analytical Model

The analysis is performed by a three dimensional and nonlinear finite element analysis program based on the large deflection theory of frame elements[v]. Geometric nonlinearity is introduced using the updated lagrangian method and the analysis is performed by incremental displacement control, which is combined with the Newton-Raphson method. The frame elements are modeled considering fiber technique in which the member cross section is divided into many cells.

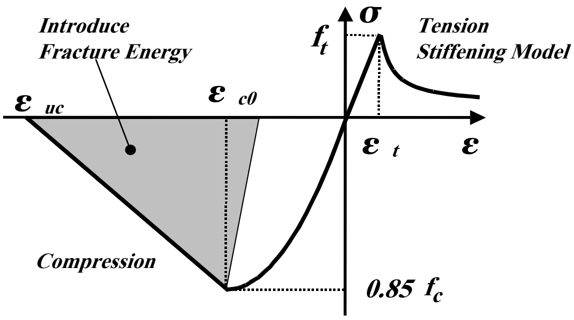

Figure 3: Stress strain relationship of concrete

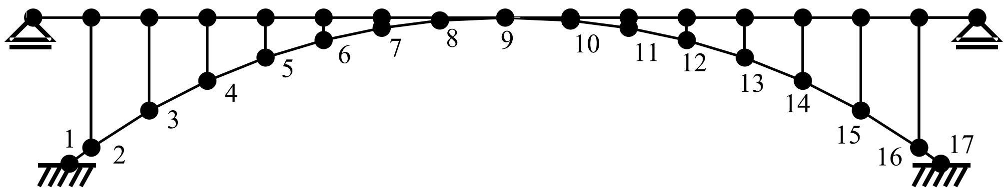

Figure 4: Analytical model

The stress strain relationship of concrete is shown in Fig.3. The softening branch in compression is determined by considering the compressive fracture energy and that in tension is modeled by considering the tension stiffening. The stress strain relationship of reinforcing bar is modeled by bi-linear model.

The 600m span concrete arch bridge is modeled as shown in Fig.4 with 33 nodes and 46 elements. The ends of the arch ribs are fixed and the vertical directions of the ends of the stiffening girders are also fixed. After the dead load is applied, the load proportional to the dead load vector corresponding to each node are applied under control in each direction of the vertical, the bridge axis and the perpendicular to the bridge axis. The stability is checked by the negative eigen value for the tangential effective stiffness matrix described in section 2.1.

### 4.2 Analytical Results

The load displacement relationship at the displacement controlled node in each direction and the changes in the eigen values of the tangential effective stiffness matrix are shown in Fig.5. In the vertical direction, the load carrying capacity is predicted about 2.55 times of the dead load. Although the deformation mode is symmetric up to near the peak load, it change to the asymmetric mode from the point marked with ▲ as shown in Fig.6. It is supposed that the predicted peak load shows the value on the bifurcation branch beyond the bifurcation point detected at the point marked with ▲. The bifurcation branch, however, does not show the rapid decrease of the load from the point and follows ascending curve up to the peak load. Therefore, the point does not influence seriously to the stability. It is noted that the displacement controlled point is node 14 in Fig.4, since the displacement at the top of the arch rib (node 9) showed the snap-back behavior after the peak due to the asymmetric deformation mode. In the case in the bridge axis direction and the direction perpendicular to the bridge axis, the displacement at the top of the arch rib (node 9 in Fig.4) is controlled. The load carrying capacities are about 0.09 times and 0.23 times of the dead load in each directions, respectively.

In all cases, the eigen values of the first and the second order of the tangential effective stiffness matrix gradually become smaller as the deformation increase. The values of the first order become zero when the maximum load is reached and show the negative values after the point. While the value of the second order is still positive values after the point. Therefore, the negative values of first order eigen value mean that the post peak range is on the fundamental path beyond the limit point which are occurred at the maximum load point. On the path, the designed arch bridge show the stable deformation behavior in post-peak range without undergoing sudden failure according with the chang of the deformation mode such as bifurcation.

## 5 STABILITY VERIFICATION UNDER SEISMIC LOADS

In this chapter, the stability in dynamic action for the designed concrete arch bridge is investigated by two kinds of the

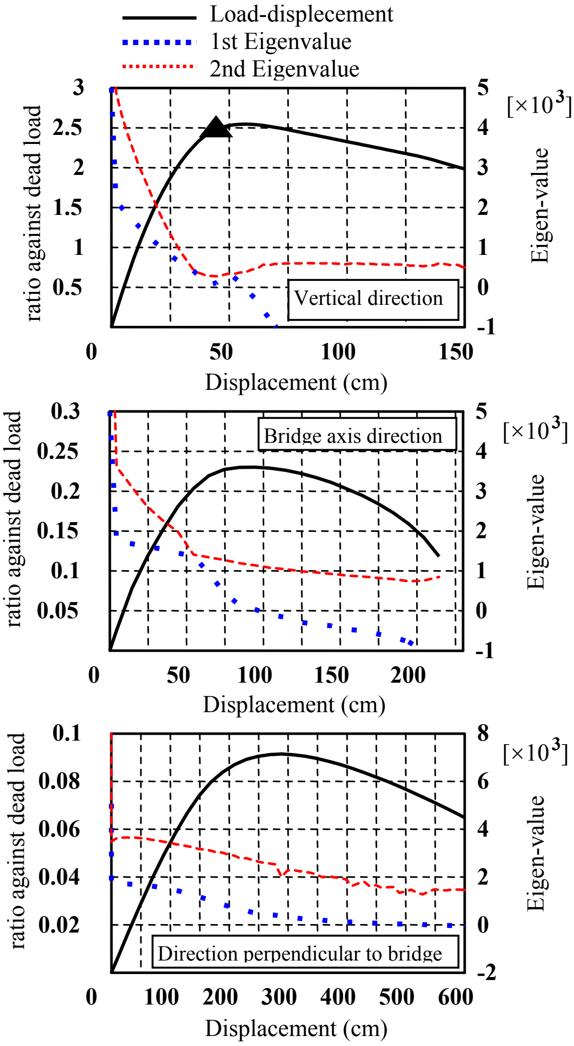


Figure 5: Load-displacement relationship and eigenvalues

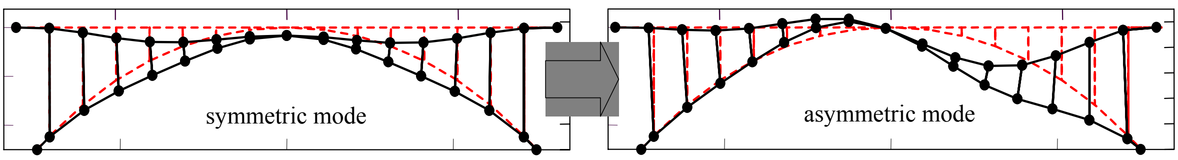


Figure 6: Change of displacement mode before and after point ▲

verification methods described in chapter 2. In the analysis, Newmark's $\beta$ method is used to integrate the equation of dynamic motion with $\beta = 0.25$ . Damping ratio of concrete members is given as 3%. The time history acceleration waveforms are used the ground motion observed at JMA Kobe in the Great Hanshin Earthquake whose maximum acceleration is 812 $cm/sec^2$ . The acceleration wave is inputed simultaneously in the three directions such as the vertical direction, the bridge axis direction and the direction perpendicular to the bridge axis.

## 5.1 Check of Negative Eigen Value Tangential Effective Stiffness Matrix

The analytical model and the analytical method is the same as the method used in static loading. Figure 7 shows the time history response displacement at the top of the arch rib for the directions of the bridge axis and the direction perpendicular to the bridge axis. Figure 8 shows the time history of the first eigen value. The maximum displacement observed in duration of the earthquake motion is 20.1cm for the direction of the bridge axis and 85.0cm for the direction perpendicular to the bridge axis. These are quite small values as compared with the displacement corresponding to the maximum load in the cases of static loadings, which are 90cm and 290cm respectively. Although the first eigen value becomes negative temporarily in an early stage, it shows the almost constant values finally. It was confirmed that these negative eigen values does not observed when the tangential stiffness of the concrete after the maximum tensile stress is set as zero. Therefore, the negative values does not influence to the stability of structural system, since they were caused numerically by the negative tangential stiffness of concrete under the tensile stress due to rapid decreases in stress. As a result, the designed 600m span concrete arch bridge does not reach the limit point (maximum load point) as well as the bifurcation point and is stable under seismic load.

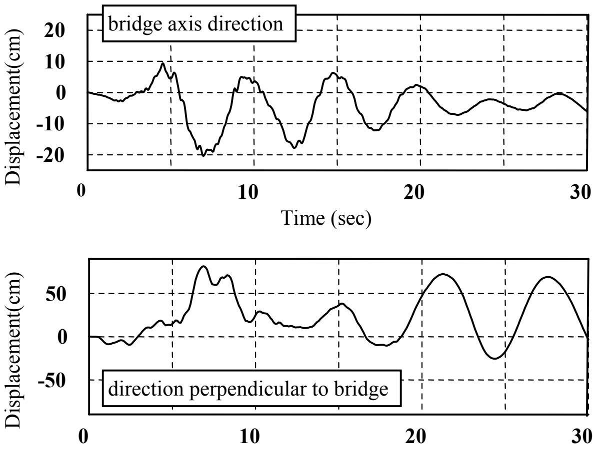


Figure 7: Responce displacement at top of arch rib

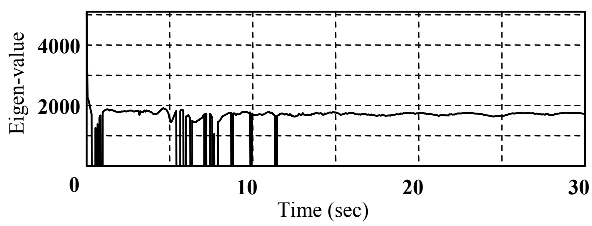


Figure 8: Time history of first eigenvalue

## 5.2 Check of Seismic-Buckling Capacity Indicators

The different analysis program as mentioned in previous section is used to check the seismic-buckling capacity indicators, though the analytical conditions are same. The program is based on the updated lagrange formulation were carried out on a three dimensional frame model, in which the moment curvature relationships of the cross sections are assumed. The moment curvature relationships of concrete arch-lib members are used degrading tri-linear hysteretic model and the other members are modeled with elastic beams. The specification of the platform program are shown in Table 2.

Table 2: Specification of the Platform Program

| | |
|---|---|
| Category | Three dimensional non-linear frame analysis program |
| In-elastic | Setting of the moment-curvature relationships in beam elements' bending |
| Geometrical stiffness matrix | Estimating from axial forces of beam or truss elements |
| Formulation of Geometrical nonlinear | Updated Lagrange formulation |
| Time history analysis | Direct integration method (Newmark- $\beta$ method) |
| Vibration eigen analysis | Sub-space method |
| Buckling eigen analysis | Inverse iteration method |

Time history of the minimum eigen value at intervals of 0.01 seconds in all duration is shown in Fig.9. The analysis process ended stably without occurring the eigen value of $0.0 \le \lambda_1 \le 1.0$. It means that buckling didn't come up in all duration. The time history of $\lambda_1$ show the row of irregular V shapes. We focus on the minimum eigen-value that is one of the lowest points of V shapes. So the envelope of the lowest points of V shapes (the dashed line in Fig.9) helps well to check the changes of the dangerousness. The minimum $\lambda_1$ was 239.9 which is generated at 5.87seconds. Fig.10 shows the paired buckling mode with the minimum $\lambda_1$ in the most dangerous time. It is an overall mode in which all vertical members are inclined to the transverse direction.

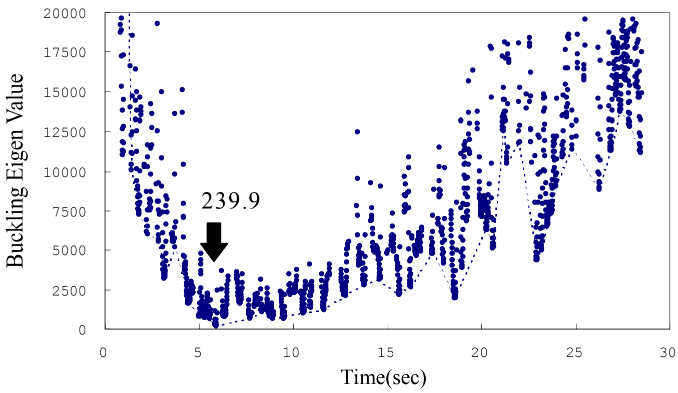


Figure 9: Time history of eigen values

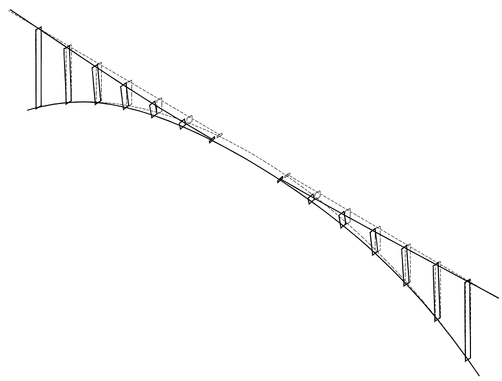

Figure 10: Buckling mode of paired with minimum eigen value

## 6 CONCLUSIONS

- Two kinds of the verification methods are presented about the stability under static and seismic loads considering the mixed (material and geometrical) non-linearlity. These methods can evaluate the stability against local or global buckling of the structural system in the analysis.

- The structural performance of a trial designed 600m span concrete arch bridge are investigated with check of the stability under the static and seismic loads. It has the sufficient seismic and stableness performance even in post-peak range without undergoing sudden failure according with change of deformation mode such as bifurcation.

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