Arch Bridges
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TIMBER ARCH BRIDGES: A DESIGN BY LEONARDO

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Abstract. *In this study structural behaviour of the timber bridge drawn by Leonardo from Vinci in Fol.22 r.a, Atlantic Code, is analyzed, which can be ascribed to a special timber arch bridge typology. In fact, the arch behavior is obtained by the spatial disposition of rectilinear joists. Based upon Leonardo's sketch, a geometric model of the military bridge has been made, focusing upon the more interesting technological aspects of this solution: timber beams connections, made resorting to special lashings; timber beams disposition on staggered planes, making a crushproof spatial whole, which behaves like an arch. In such reconstruction, the technological abilities of the epoch in which the bridge would have had to be built are kept in mind. At this aim, the study of the Palladian reconstruction of Caesar's bridge on the Rhine is particularly meaningful, in relation specifically to connections with lashings. With reference to the disposition of timber beams, it is of great interest the comparison with the bridge built in 2000 at Jinze, in the outskirts of Shanghais, as a reproduction of the ancient "rainbow bridge ", built during the Sung Dynasty (960-1280 A.D.) in the city of Khaifeng.*

The hypotheses on which the model is based have been verified with opportune schemes of calculation which confirm the validity of the structural conception conceived by Leonardo. Insofar Leonardo's military bridge constitutes a meaningful step in the evolutionary path of the timber arch bridge made with rectilinear elements, a path which can be considered to start with Traiano's bridge on Danube river, and to end with the last great Swiss bridges of the eighteenth century.

1 INTRODUCTION

All those bridge typologies, whose structural behaviour is referable to the arch, can be considered as arch bridges. Since timber has always shown notable difficulties to assume curved forms, which being unnatural are not proper for it, many bridges builders, recognizing the effectiveness of the structural arch scheme, have tried to evade the obstacle through clever combinations of single timber elements of quite small dimensions.

At the age of Leonardo, indeed, attempts to realize timber curved beams were already made, and he deals with this problem in some early papers of technical argument in the Atlantic Code, which can be dated to the last decades of fifteenth century. In Fol.33 v.b, for instance, there is the drawing of a trestle which, by mean of a setscrew, bents a toothed timber beam. More Leonardo's drawings on the same subject can be found in other papers of the same code, Fol.33 r.a and Fol.344 v.a, giving two different models of the curving trestle[i].

An interesting proposal is also contained in the "Machinae Novae" by Veranzio from Sebenico (Venice, 1615), relating to an arch timber bridge, made with toothed curved beams.

From eighteenth century onward different systems for making timber arches using tables or joists curved and jointed with various methods were conceived, until modern glued laminated timber structures appeared.

In Leonardo's work a special interest in designing provisional timber bridges can be noticed, especially in the Atlantic Code, all intended for military use. In fact, in the famous letter addressed to Ludovico Sforza in 1482, which preludes to his moving at the Court of Milan, Leonardo, suggesting himself as an expert in military designing of structures and machines, affirms to have designs of very light and tough bridges suited to run after, and sometimes away, the enemies, as well as safe and fireproof bridges, easy to build and then to dismantle[ii].

The timber bridge structural typology examined in this study is that one in which the arch behaviour is obtained thanks to a wise assemblage of timber rods. In fact the military bridge which Leonardo draws in Fol.22 r.a. of Atlantic Code, is an arch structure made with rectilinear beam jointed with ropes. The interest for this structure lays in all of these two aspects: arch behaviour of the assemblage of straight elements, and joints peculiarity.

1.1 Arch bridges made by joists of small dimensions

The first survey of an arch bridge belonging to this typology is the timber bridge which Emperor Traiano ordered to build between 103 and 105 a.C. on Danube river, during the wars against the people of Dacia, and which was designed by Apollodoro from Damasco, as quoted by the proconsul Dione Cassio in his "Historia Romana". This bridge is sketched on the Traiano's Column in Rome, and, as proved by the surveys carried out in the riverbed at the aim of identifying the masonry piers, it was constituted of twenty-one arches with a clean span of about 32 m.

Then only later on, in the famous treatise of Andrea Palladio "I quattro libri dell'architettura", Venice, 1570, it is possible to find a timber arch bridge made with little joists, between his "inventions" for building bridges without intermediate piers in the riverbed. Referring to the arch abutments it could be inferred that the arch extrados ought to be the walkway[iii] or, as in the hypothesis of Zorzi[iv], Palladio designed it as a timber centring of a masonry arch bridge. The hypothesis of a centring introduces the figure of Jean Rodolphe Perronet , who, in his work “Memoire sur le cintrement et le decintrement des ponts”, 1792, devoted to the realization technique of large span masonry arch bridges, designed a new system of polygonal centring[v], employed afterwards as a proper timber structure in the bridge of the Salpetriere. M. Gauthey, in his treatise[vi], states that this system was frequently used in France and, as an example, reports the bridge of the Mulatieré, built at Lion on the Soane.

This timber bridge typology was also developed in England, as is testified by the very famous Old Walton Bridge, built on the Thames in the Surrey, by William Etheridge, in 1750, also reproduced in a picture by Canaletto, dated to 1754, kept in the Dulwich Picture Gallery.

In any case it is universally accepted that the timber bridge typology with polygonal bearing structures greatly developed in Switzerland[vii], reaching the highest levels, as in the Hundwilertobel bridge, near Zurich, built in 1778 by Hans Ulrich Grubenmann, with a clean span of 30,00 m. These Swiss bridges with a polygonal frame haven’t the same structural behaviour of the other bridges previously described, as the polygonal structure is inserted into a box-like ensemble which, together with lower beams and upper covering system, jointed by double hanging posts, forms a structural whole of great rigidity. Even if an arch behaviour can be ascribed to the system constituted by the straight elements of the polygonal frame, locked by the hanging posts, the whole complex behaves as a bending beam.

2 LEONARDO’S ARCH BRIDGE

In Fol.17 v.a e 22 r.a of the Atlantic Code, dated to 1487-1488, (Fig.1), together with two sketches of an arbalest, and the drawing of a church inspired to Florence Cathedral, three studies of timber bridges can be seen: a bridge on trestles, a bridge described by Leonardo as an easily building one, and the military bridge which is the argument of the present note.

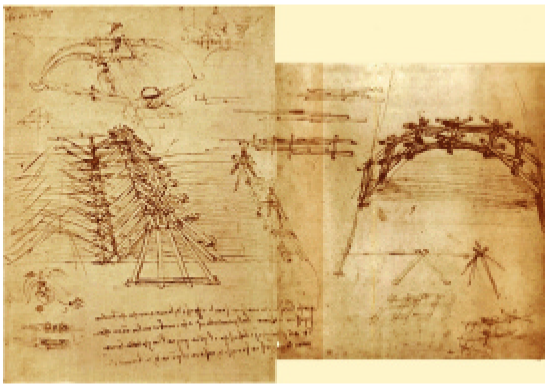

Figure 1: Fol. 17 v.a, on the left, and Fol.22 r.a, on the right, Atlantic Code.

The last one is constituted by two parallel frames, with an arch shape, transversally connected by horizontal joists. Each frame is formed with straight rods of circular cross-section, disposed on staggered planes and connected by lashings, close at the transversal joists. The same bridge is also drawn in Fol.23 v.a., dated to 1487-90, nearby some studies on artillery (Fig. 2).

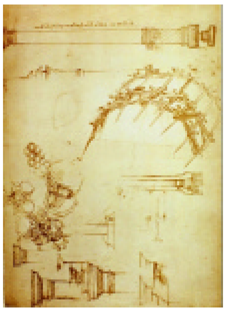

Figure 2: Fol.23 r.a, Atlantic Code.

Comparing the two sketches reveals some differences: the first bridge has a lowered curve, with the arches formed by five rods each one; the second one has a round curve with the arches formed by nine rods. Moreover while the side support in the first scheme is given by a single rod, in the second one two rods are inserted. Anyway, both solutions are referable to the same geometrical configuration: the partition of the circular sector in equal parts allows locating the extremities of the chords, which correspond to the rods alternately disposed, in the points of intersection of the radii with the circumference (Fig.3).

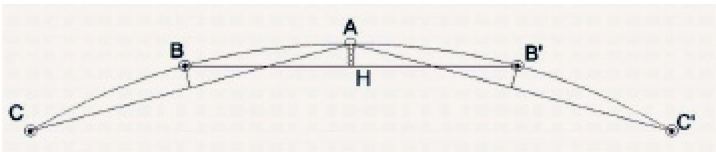

Figure 3: Geometrical genesis of military bridge.

A deeper study on the geometry of the bridge has allowed to infer that the configuration of each arch depends upon four parameters: span, rise, number of parts in the circular sector and diameter of the transversal section of the rods; and only three of these are independent. For instance, once the span and the number of divisions have been fixed, enlarging the rods diameter induces an increase in the arch rise.

2.1 The problem of the joints

The spatial configuration of each arch involves the disposition of the rods staggered on three parallel planes, jointed nearby the transversal joists, where the rods come along-side (Fig.4).

The constraint imposed by each lashing allows the mutual rotation between the rods, but, looking at the basic constructive modulus (Fig.5), whose iteration generates the whole arch structure, it can be noticed how the mutual rotation of two inclined rods is prevented by the contrast generated by the transversal joists. In the lower part of Fig.5 is reproduced the deformation diagram of the structural scheme of the constructive modulus, in which the presence of transversal joists is schematized with vertical hinged rods of the same stiffness, and loads are modelled by forces applied at the free ends of the inclined rods. It is evident that the efficiency of this mechanism is influenced by the lashing as the sliding of the transversal joist along the inclined rods must be prevented; to this sliding is opposing also friction between the contact surfaces, as larger as smaller is the slope of the rods.

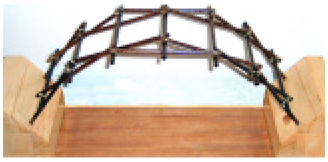

Figure 4: Timber model of the military bridge.

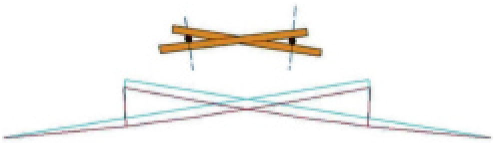

Figure 5: Basic constructive modulus of military bridge.

In Fol. 17 v. many drawings illustrating various systems of jointing round rods by lashings (Fig.6) are reproduced, together with Leonardo's notes putting in evidence the efficiency of these joints depending upon the angle of width between the rods.

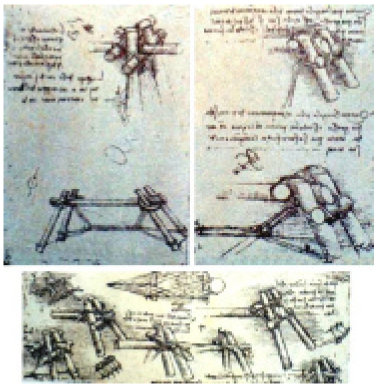

Figure 6: Fol. 17 v., Atlantic Code.

In particular, referring to Fol.17 v.d (upper image on the right in Fig.6), Leonardo writes, in the upper part, that more the width of the angle is, less is the strength of the joint, if the joint is fixed with nails, but, at the bottom he writes that more this width is, more the junction of the rods is strong, if lashing are used instead. In all these sketches, the angle between the main bearing elements is definitely lesser than that in the basic modulus of the military bridge (Fig.5); it more resembles the angle which can be seen in the trestles bridge of Fol.17 v.a. (Fig.1, on the left), whose structural scheme (Fig.7) can be traced back, apart from the angle width, to the basic modulus (Fig.5) by a simple rotation. The less angle width in the trestles bridge induces, together with an enlargement of the sliding for the transversal joists, also their moving closer, if an admissible diameter must be assumed for their cross section. This different disposition causes a consistent increase of the radial compressive stress to which the transversal joist undergoes, while the flexional behaviour of the inclined rods is completely the same, apart from an increase of the maximum bending moment (Fig.8).

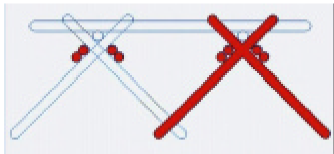

Figure 7: Structural scheme of trastles bridge.

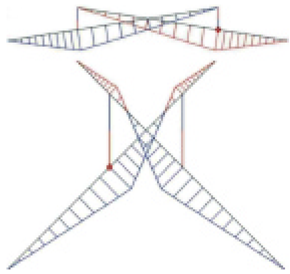

Figure 8: Moment diagrams for the two differerent structural modulus.

Looking more closely at the joints with lashings (Fig.6) brings in mind the hypothesis advanced in relation to the bridge built in 55 B.C. by Julius Caesar, described in his "Bellum Gallicum", IV, 17. Caesar's description gave rise to a great interest in the architects and humanists of the fifteenth and sixteenth centuries, who considered his bridge a paradigmatic example of timber bridge constructive technique. As the Latin text showed many interpretative difficulties, there were various reconstruction hypothesis of Caesar's bridge, often very different from each other from a structural point of view. The dispute concerned, above all, the interpretation of the word "fibula", which refers to the connection between the upper ends of the inclined rods forming the trestles, as a nailed or dowelled joint, or as a lashing, or also as a locking with timber elements. The hypothesis unanimously considered as the most plausible[viii] is that proposed by Palladio in Chapter III of his treatise, devoted to timber bridges, and also in the edition of Caesar's "Commentaria" which he illustrated in 1574. The Palladio's interpretation for the "fibulae" is that they are realized with two transversal timber joists, disposed in notches arranged in the upper part of the inclined rods, alternate on the opposite sides of them, and spaced by the transversal beam of the trestle. The function of these joists is that of locking together the inclined rods and the cross-beam constituting the trestle, keeping the distance between these elements fixed. In this way the lashed joints ought to fasten more strongly with the assessment of the piers of the trestle, so as Caesar himself described. Howard Burns brings the attention, more than on these known drawings of Palladio, on a sketch of the detail of the junction system between cross-beam and inclined rod, drawn in a table, kept at the Royal Institute of British Architects in London, related to the Nerva's Forum. In this sketch (Fig.9) two different hypothesis are superimposed, traced with a different texture: the lighter drawing shows the hypothesis in which the cross-beam and the inclined rod meet at a quite right angle, and are connected by two pins; the second drawing, reproducing the Palladio's hypothesis as quoted before, is traced over the first and thicker, and so can be seen as a subsequent correction. Burns thinks that perhaps Palladio, while illustrating to an unknown interlocutor his ideas about the working way of Caesar's bridge, first took the solutions with pins, rectifying it next with his final hypothesis. It is interesting to make a comparison between this sketch and Leonardo's words about his drawings in Fol.17 v.d (Fig.6) in relation to the larger width of the angle between inclined rods and cross-beam as well as referring to the lashings instead of nailed joints. From a structural point of view, the behaviour of the joint, assumed by Palladio for the Caesar's bridge, seems to have been already very clear to Leonardo, as sketches and notes in Fol.17 v prove.

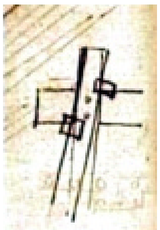

Figure 9: Palladio's sketch of Caesar's bridge joint.

In conclusion, the structural behaviour of the trestles bridge (Fig.1, on the left), as well as that of the military arch bridge (Fig.1, on the right), is based on the same working principle of the joint between main structural elements, principle to which is also due the stability of Caesar's bridge, in the Palladio's reconstruction.

2.2 Structural behaviour of the military arch bridge

To study the structural behaviour of the military bridge, a geometrical scheme has been assumed, obtained fixing a span of 15,00 m, with a partition of the arch in eight parts, with rods of circular cross-section with a diameter of 0.15 m. Calculus has been done with a FEM program (Nolian by Softing), in linear elastic range, on a spatial scheme, made of mono-dimensional hinged beams, in which mutual contact condition between staggered surfaces has been simulated with rods of adequate stiffness. Overloads, concentrated on the bond, take into account pedestrian use of the bridge. Both the solutions, with only one end rod in the side supports, as in the sketch of Fol.22 v.a (Fig.1), and with two rods, as in Fol.23 r.a (Fig.2) have been analyzed. In any joint of both solution four rods converge, with the exception of the joints near the end supports of the first scheme, where the joists are three. This circumstance induces a different structural behaviour, with a reduction to almost an half of the bending moment in the rods near the end supports, thanks to the addition of the rods in the second scheme. With reference to normal stress, instead, the added rods take on a little amount of this stress. The arch behaviour is quite evident from the comparison between the normal stress diagrams referring to the theoretical arch on which the geometrical construction of the military bridge is based (Fig.10a) and to the two frames making the arch of the structural scheme of Leonardo's bridge (Fig.10b), respectively. The two schemes show a very good coincidence of normal stress values in the key and end rods, while for the two intermediate rods this coincidence there is in the mean value of stresses, instead of in the value of each rod. In any case, in both the schemes the stress curve is sensibly far from the geometrical axis of the structure, as the bending effect is quite relevant.

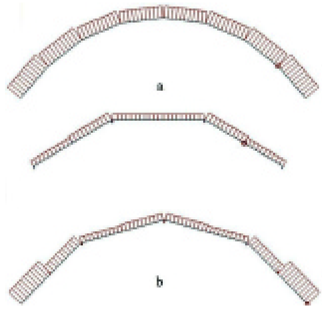

Figure 10: Normal stress diagrams.

3 RAIBOW BRIDGE

An extraordinary resemblance with Leonardo's drawing of the military bridge is shown by an humpbacked bridge built in China in twelfth century, during the hegemony of Song Dynasty, called Rainbow Bridge.

This bridge is reproduced in the famous scroll Chhing-Ming Shang Ho Thu (Going on the river to the spring Festival), ascribed to Chang Tse-Tuan and taken in the Silk Museum of Beijing, (Fig.11), which describes scenes of daily life at Khaifeng, capital city of the North Sung Empire. The building typology to which the name "rainbow bridge" is associated had large diffusion in eleventh and twelfth centuries, and was originated by the need of avoiding piers in the riverbed which obstructed fluvial navigation, the principal commercial way in an epoch of great economical development[ix,x].

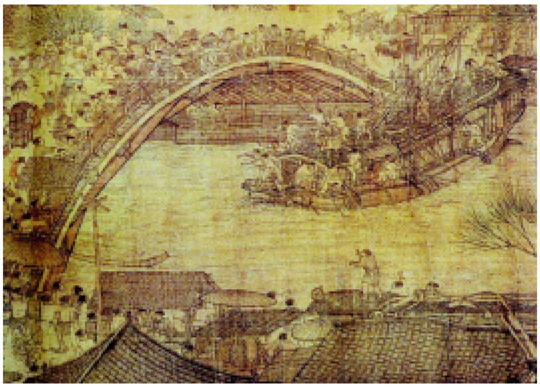

Figure 11: A detail of the Chinise skroll.

The interpretation of the structural behaviour of Rainbow Bridge has been done basing upon the rebuilding made by Prof. Bashar Altabba, on behalf of NOVA Television Company, in 2000, at Jinze, in the outskirts of Shanghais, on a ship-canal connected to Yangtze River[xi].

This rebuilding has been carried on with the help of experts in the history of architecture and technology of the Chinese imperial age.

The geometrical matrix of this bridge is an arch of circumference with a radius of about 9,00 m, of 96° angle width, partitioned in six equal parts; the rods are disposed tangential to the arch in the points of intersection of the six radii. The bearing structure is constituted by an interlacement of sixteen frames, made of rods with circular cross-section with a diameter of 0.18 m, which show to different typologies: they are constituted of three or four rods, respectively, and are disposed alternated with a-b-b-a rhythm (Fig.12). The assemblage of these frames is conditioned by the insertion of five transversal joists, disposed at the upper end of the radii, which works as spacers between the two series of frames, giving to the whole structure an arch configuration (Fig.13).

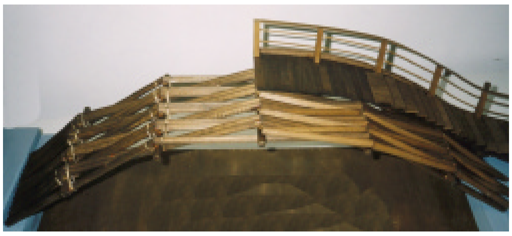

Figure 12: Timber model of Rainbow Bridge.

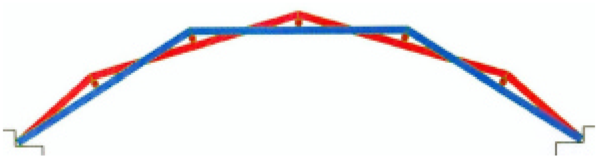

Figure 13: Schematic rapresentation of Rainbow Bridge structure.

3.1 Structural behaviour of Rainbow Bridge

Rainbow Bridge has the single rods constituting the frames disposed in the same planes, in this differing from Leonardo's bridge, in which rods compounding the two bearing arches lay on staggered planes. So, in spite of the strong shape analogy, the connections between the rods, in the Rainbow Bridge, are of "head to head" kind (Fig.13), and this constructive detail induces a different role for the joints in respect of an identical structural behaviour of the whole. Also in this bridge connections are made by lashings, in this case bamboo ropes; but, unlike Leonardo's bridge, where lashings are essential to prevent mutual sliding between inclined rods and transversal joists, in Rainbow Bridge these lashings have the only function of making stable the complex of rods. The same function is also granted by the close approaching of the frames, which resembles the warp of those baskets made of vegetable interlaced fibres. In addition to the structure above described, the bridge shows joists supporting the deck, with the same cross-section of the others, which lean diagonally on the transversal joists, and are disposed at the joints (Fig.14).

Numerical analysis of Rainbow Bridge has been done with the same procedure followed for the military bridge. Results show a quite uniform distribution of normal stress in all the frames, with three or four rods (Fig.15), reaching substantially equal maximum values.

Figure 14: Secondary structure of Rainbow Bridge.

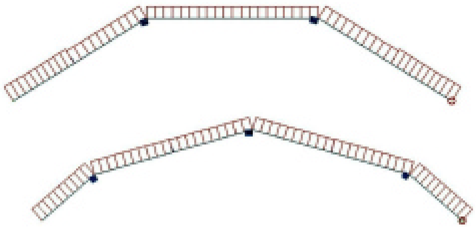

Figure 15: Normal stress diagrams of the two frames.

Bending moment values are very small, so that eccentricity of stresses in the rods is very little, quite everywhere inside inertia central kernel. In conclusion, the disposition of rods employed in the Rainbow Bridge, if compared to the solution deduced by Leonardo's sketch, implies a reduction of bending effects and consequently a more close arch behaviour.

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